Circle diagram at no load (Φr=0)

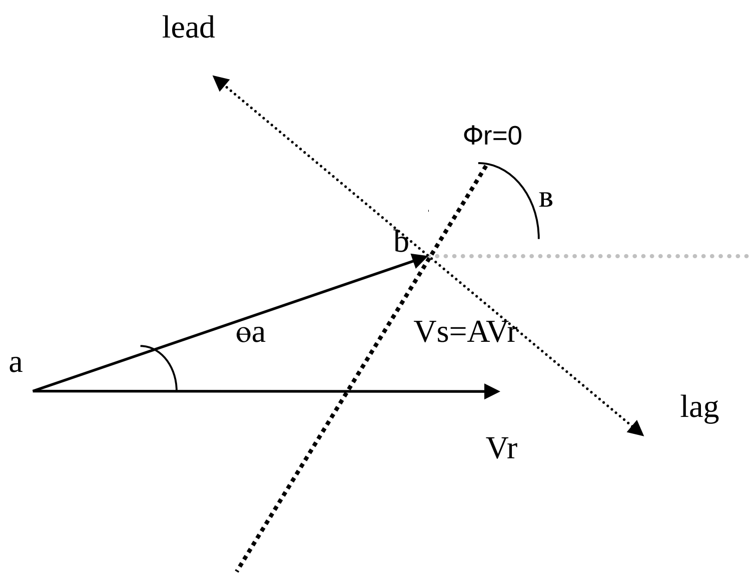
$$I_r = 0 \qquad \phi_r = 0$$

$$V_s\angle\theta = AV_r\angle\theta_a + BI_r\angle\beta \pm \phi_r$$

$$V_s\angle\theta = AV_r\angle\theta_a$$

$$\theta = \theta_a$$

$$\overline{ab} = |V_s| = |AV_r|$$



Dr. Eng: M. A. Ebrahim

Circle diagram at Lagging P.f (Φr= -ve value)

$$V_s \angle \theta = A V_r \angle \theta_a + B I_r \angle \beta - \phi_r$$

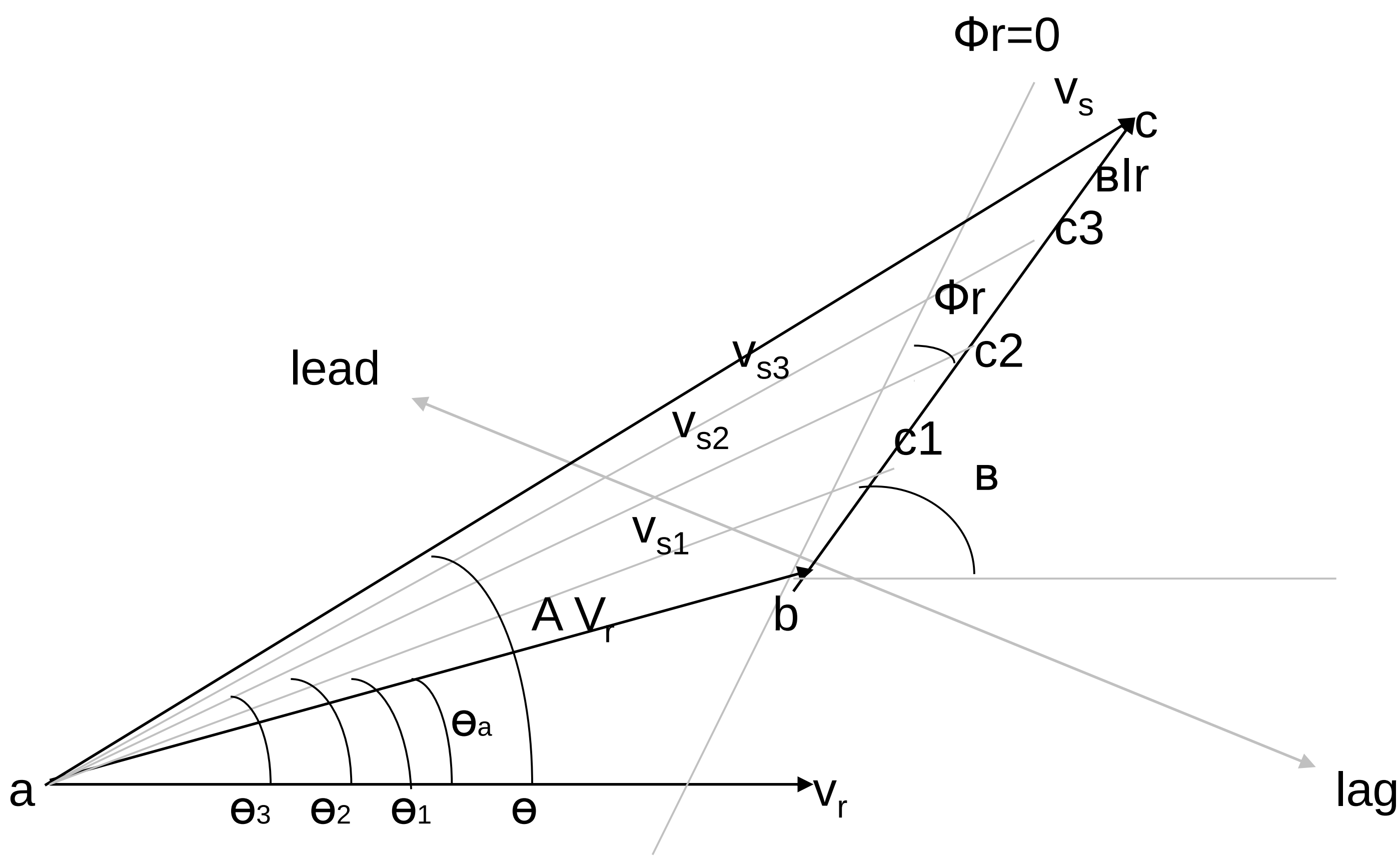
$\overline{ac_1} = V_{s1}$ at $\frac{1}{4}$ load

$\overline{ac_2} = V_{s2}$ at $\frac{1}{2}$ load

$\overline{ac_3} = V_{s3}$ at $\frac{3}{4}$ load

$\overline{ac} = V_S$ at full load

Continue



Φr=0
vs
c
ʙIr
c3
Φr
c2
lead
vs3
vs2
c1
ʙ
vs1
A Vr
b
ɵa
a
ɵ3
ɵ2
ɵ1
ɵ
vr
lag

Circle diagram at Leading P.f (Φr= +ve value)

$$V_s \angle \theta = AV_r \angle \theta_a + BI_r \angle \beta + \phi_r$$

$$\overline{bc} = BI_r$$

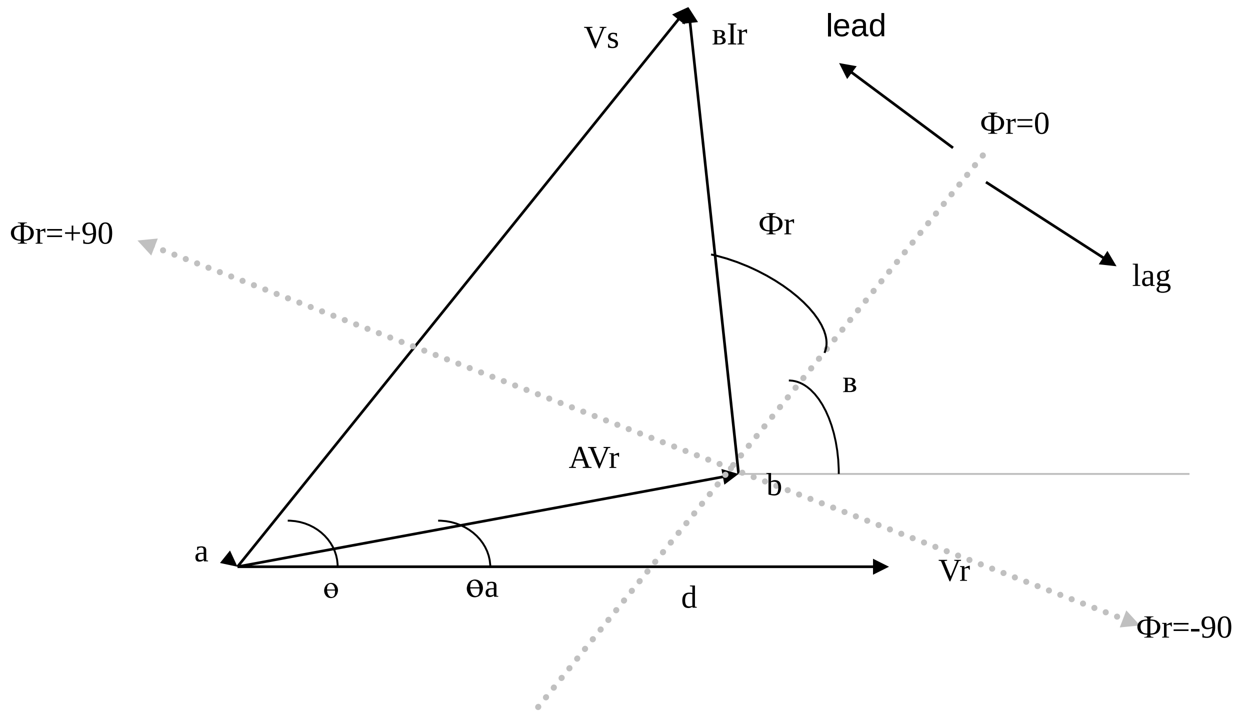
$$\overline{ac} = V_s$$

$$c\widehat{a}d = \theta$$

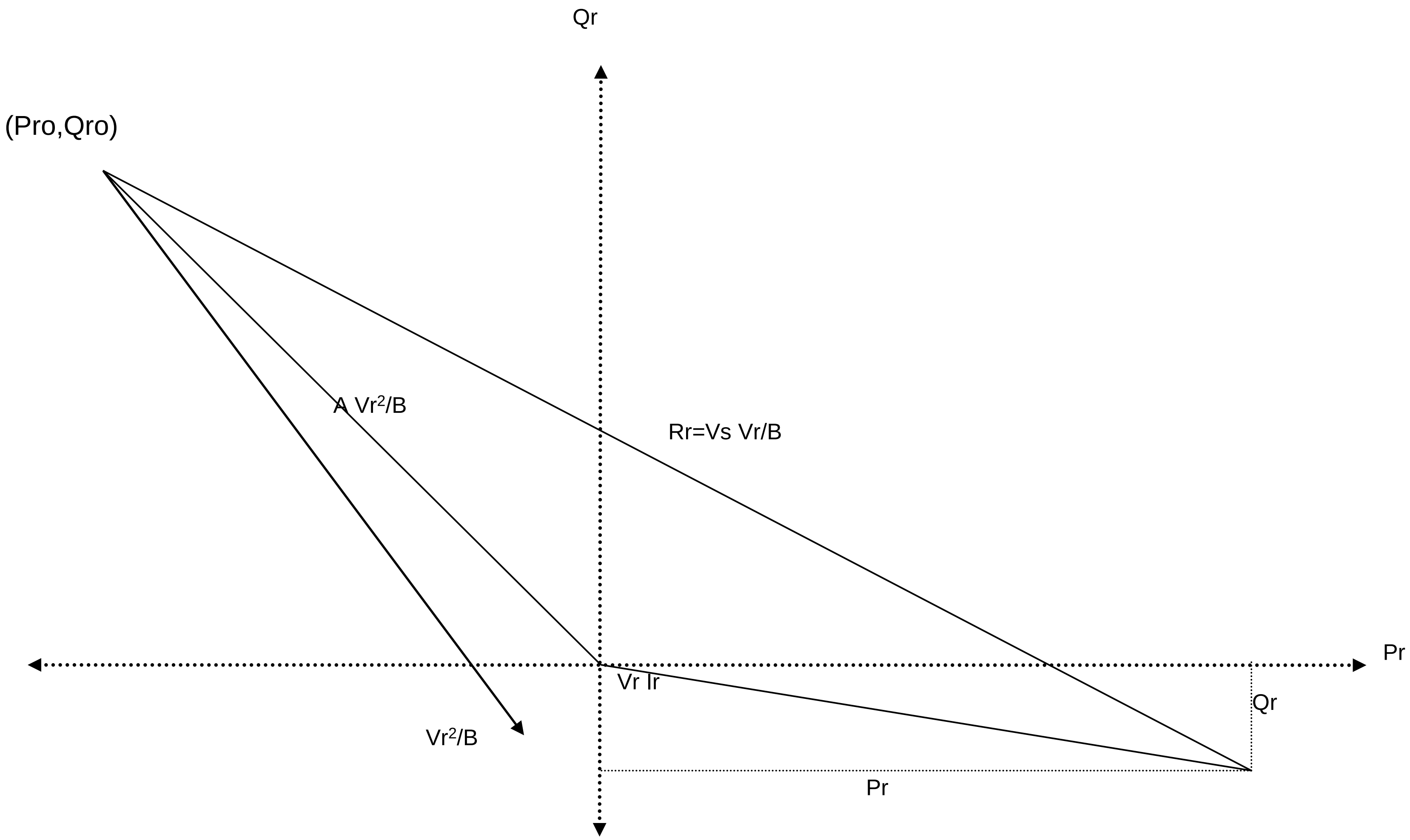
$$\overline{cD} = BI_r \sin \phi_r$$

$$\overline{bD} = BI_r \cos \phi_r$$

Continue



Receiving-End Power Circle Diagram



Dr. Eng: M. A. Ebrahim

Continue

$$V_s = AV_r + BI_r \qquad I_r = \frac{V_s}{B} - \frac{AV_r}{B}$$

$$I_r^* = \frac{V_s^*}{B^*} - \frac{A^* V_r^*}{B^*}$$

$$P_r + jQ_r = V_r I_r^*$$

$$P_r + jQ_r = \left(-\frac{A^* V_r^*}{B^*} \right) V_r + \left(\frac{V_s^*}{B^*} \right) V_r$$

$$P_r + jQ_r = -\frac{A^*}{B^*} \left| V_r \right|^2 + \frac{\left| V_r V_s \right|}{B^*} e^{-j\theta}$$

Continue

$$P_{ro} = -\frac{AV_r^{\,2}}{B}\cos(\beta - \theta_a)$$

$$Q_{ro} = +\frac{AV_r^{\,2}}{B}\sin(\beta - \theta_a)$$

$$P_{ro} = -\frac{AV_r^{\,2}}{B}(\cos\beta\cos\theta_a + \sin\beta\sin\theta_a)$$

$$Q_{ro} = \frac{AV_r^{\,2}}{B}(\sin\beta\cos\theta_a - \cos\beta\sin\theta_a)$$

Continue

assume,

$$A = a_1 + ja_2 = A\angle\theta_a \qquad , B = b_1 + jb_2 = B\angle\beta$$

$$\sin\theta_a = \frac{a_2}{A} \qquad \cos\theta_a = \frac{a_1}{A}$$

$$\sin\beta = \frac{b_2}{B} \qquad \cos\beta = \frac{b_1}{B}$$

$$Q_{ro} = \frac{+AV_r^2}{B}\left[\frac{b_2a_1}{AB} - \frac{b_1a_2}{AB}\right] = \frac{+V_r^2}{B^2}\left[a_1b_2 - a_2b_1\right]$$

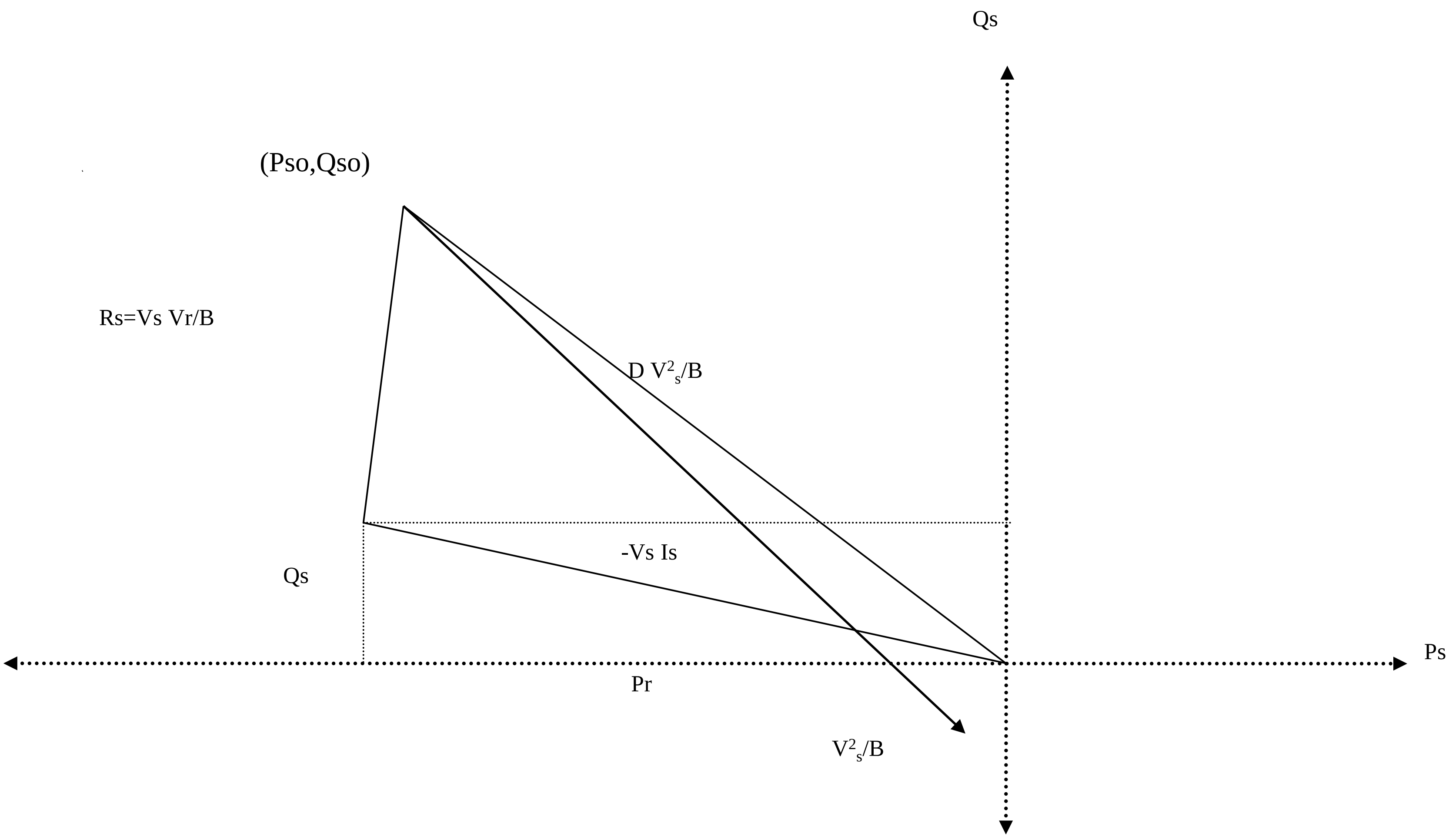
$$P_{ro} = \frac{-AV_r^2}{B}\left[\frac{b_1a_1}{AB} + \frac{b_2a_2}{AB}\right] = \frac{-V_r^2}{B^2}\left[a_1b_1 + a_2b_2\right]$$

Continue

$$\left(P_r - P_{ro}\right)^2 + \left(Q_r - Q_{ro}\right)^2 = R_r^{\,2}$$

$$\left(P_r + \frac{a_1 b_1 + a_2 b_2}{b_1^{\,2} + b_2^{\,2}} V_r^{\,2}\right)^2 + \left(Q_r - \frac{\left(a_1 b_2 - a_2 b_1\right)}{b_1^{\,2} + b_2^{\,2}} V_r^{\,2}\right)^2 = \frac{V_s^{\,2} V_r^{\,2}}{b_1^{\,2} + b_2^{\,2}}$$

Sending End Power Circle Diagram



Continue

$$DV_s - BI_s = DAV_r - BCV_r + BDI_r - BDI_r$$

$$= \left(DA - BC\right) = V_r$$

$$\overline{oa} = \frac{DV_s^2}{B}$$

$$\overline{ab} = -\frac{BI_sV_s}{B} = -I_sV_s$$

$$\overline{ob} = \frac{V_sV_r}{B} = R_s$$

Continue

$$\overline{al} = I_s V_s \cos\phi_s = P_s$$

$$\overline{bl} = V_s I_s \sin\phi_s = Q_s$$

$$P_{so} = \overline{om} = \frac{D V_s^{\,2}}{B}\cos(\beta - \theta_a)$$

$$Q_{so} = \overline{on} = \frac{D V_s^{\,2}}{B}\sin(\beta - \theta_a)$$

Continue

$$V_r = DV_s - BI_s$$

$$= (d_1 + jd_2)V_s - (I_{sp} + jI_{sq})(b_1 + jb_2)$$

$$V_r = (d_1V_s - b_1I_{sp} + b_2I_{sq}) + j(d_2V_s - b_2I_{sp} - b_1I_{sq})$$

$$V_r^2 = (d_1^2 + d_2^2)V_s^2 + (b_1^2 + b_2^2)(I_{SP}^2 + I_{SQ}^2)$$

$$- 2(d_1b_1 + d_2b_2)V_sI_{sp} - 2(d_2b_1 - d_1b_2)V_sI_{sq}$$

Continue

$$\frac{Vr^2}{b1^2+b2^2}=\frac{d1^2+d2^2}{b1^2+b2^2}Vs^2$$

$$+\left[I_{sp}^2-2\frac{(d1b1+d2b2)}{b1^2+b2^2}V_s I_{sp}\right]+\left[I_{sq}^2-2\frac{(d_2b_1-d_1b_2)}{b_1^2+b_2^2}V_s I_{sq}\right]$$

$$\frac{V_r^2 V_s^2}{b_1^2+b_2^2}=\left[I_{sp}V_s-\frac{(d_1b_1+d_2b_2)}{b_1^2+b_2^2}V_s^2\right]^2$$

$$+\left[I_{sq}V_s+\frac{(-d_2b_1+d_1b_2)}{b_1^2+b_2^2}V_s^2\right]^2$$

Continue

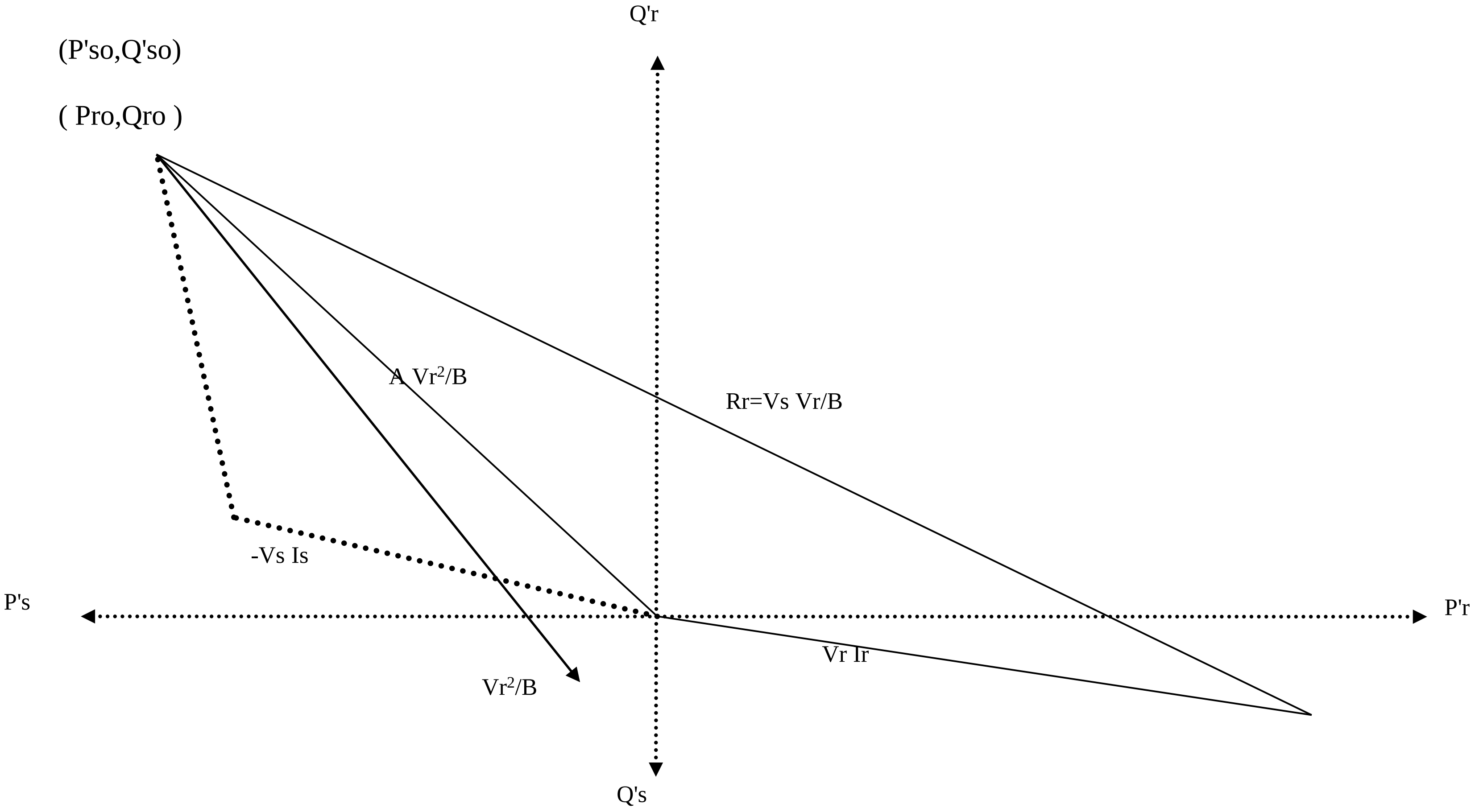
$$R_s^2 = \left(P_s - P_{so}\right)^2 + \left(Q_s - Q_{so}\right)^2$$

$$P_{so} = +\frac{\left(d_1 b_1 + d_2 b_2\right)}{b_1^2 + b_2^2} V_s^2$$

$$Q_{so} = -\frac{\left(d_2 b_1 + d_1 b_2\right)}{b_1^2 + b_2^2} V_s^2$$

$$R_s = \frac{V_s V_r}{B}$$

The Universal Power Circle Diagram



Continue

from R.E. PCD:

$$\left[P_r + \frac{a_1 b_1 + a_2 b_2}{b_1^2 + b_2^2} V_r^2\right]^2 + \left[Q_r - \frac{(a_1 b_2 - a_2 b_1)}{b_1^2 + b_2^2} V_r^2\right]^2 = \frac{V_s^2 V_r^2}{B^2}$$

from S.E. PCD:

$$\left[P_s - \frac{d_1 b_1 + d_2 b_2}{b_1^2 + b_2^2} V_s^2\right]^2 + \left[Q_s + \frac{(d_1 b_2 - d_2 b_1)}{b_1^2 + b_2^2} V_s^2\right]^2 = \frac{V_s^2 V_r^2}{B^2}$$

$$\left[P_r \left(\frac{V_b}{V_r}\right)^2 + \frac{a_1 b_1 + a_2 b_2}{b_1^2 + b_2^2} V_b^2\right]^2 + \left[Q_r \left(\frac{V_b}{V_r}\right)^2 - \frac{(a_1 b_2 - a_2 b_1)}{b_1^2 + b_2^2} V_b^2\right]^2 = \frac{V_s^2 V_b^4}{V_r^2 B^2}$$

Continue

$$\left[P_s\left(\frac{V_b}{V_s}\right)^2-\frac{a_1b_1+a_2b_2}{b_1^{\,2}+b_2^{\,2}}V_b^{\,2}\right]^2+\left[Q_s\left(\frac{V_b}{V_s}\right)^2+\frac{\left(a_1b_2-a_2b_1\right)}{b_1^{\,2}+b_2^{\,2}}V_b^{\,2}\right]^2$$

$$=\frac{V_b^{\,4}V_r^{\,2}}{V_s^{\,2}B^2}$$

$$\left(\bar{\mathrm{P}}_{\mathrm{r}}+\bar{\mathrm{P}}_{\mathrm{r}\,o}\right)^2+\left(\bar{Q}_r-\bar{Q}_{ro}\right)^2=\bar{R}_r^{\,2}$$

$$\left(\bar{P}_s-\bar{P}_{so}\right)^2+\left(\bar{Q}_s+\bar{Q}_{so}\right)^2=\bar{R}_s^{\,2}$$

Continue

It is noted that :

$$\overline{P}_{ro} = -\overline{P}_{so} = \frac{a_1 b_1 + a_2 b_2}{b_1^{\,2} + b_2^{\,2}} V_b^{\,2}$$

$$\overline{Q}_{ro} = -\overline{Q}_{so} = \frac{a_1 b_2 - a_2 b_1}{b_1^{\,2} + b_2^{\,2}} V_b^{\,2}$$

$$\overline{R}_r = \frac{V_s V_b^{\,2}}{V_r B} \qquad \overline{R}_s = \frac{V_r V_b^{\,2}}{V_s B}$$

$$\overline{P}_r = P_r \left(\frac{V_b}{V_r} \right)^2 \qquad \overline{Q}_r = Q_r \left(\frac{V_b}{V_r} \right)^2$$

$$\overline{P}_s = P_s \left(\frac{V_b}{V_s} \right)^2 \qquad \overline{Q}_s = Q_s \left(\frac{V_b}{V_s} \right)^2$$